

# Introducing the MAC3 Equity Factor Models

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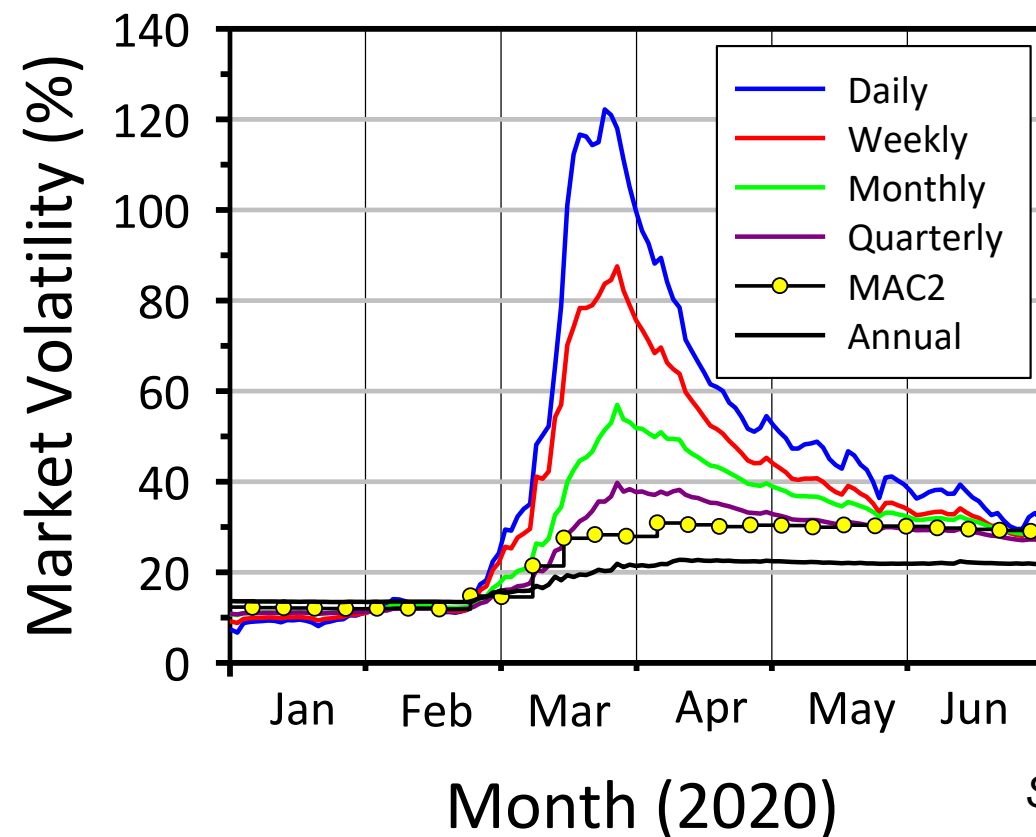
# MAC3: A Brand New Risk Model

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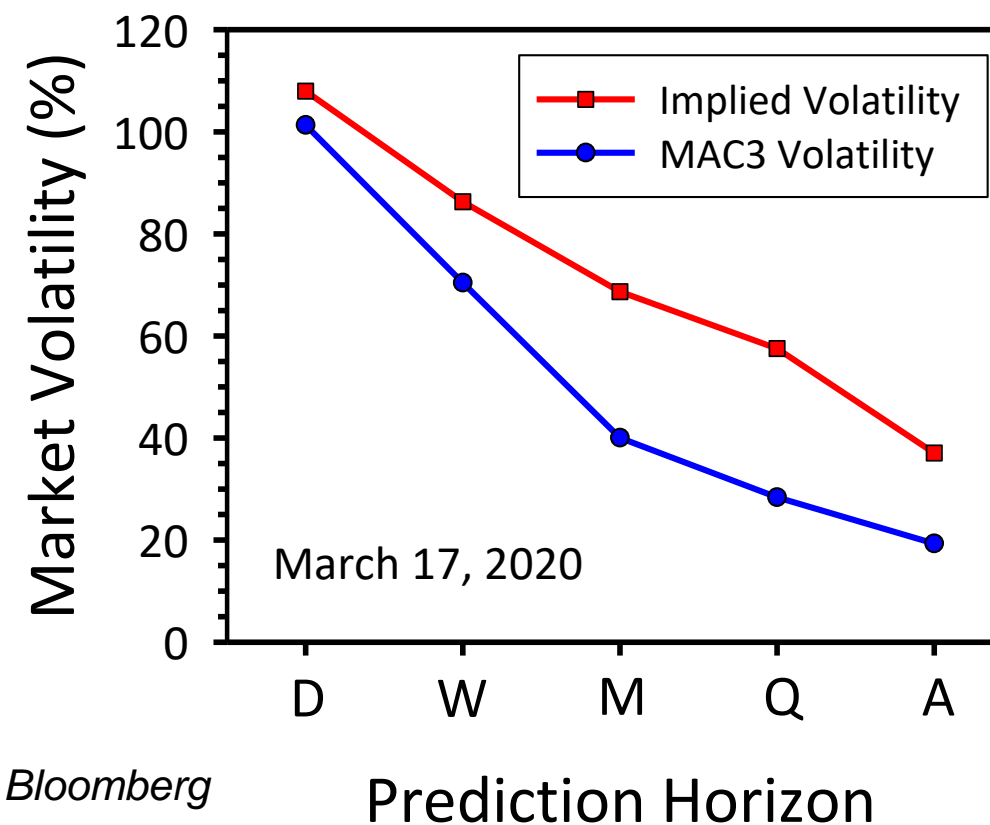
- Local focus, extended coverage
  - Example: New UK local model covers all DM European stocks
- Daily updated model highly responsive to market conditions
  - Cross-sectional observations allow volatilities to adapt quickly
- Models calibrated to six prediction horizons (D, W, M, Q, A, LT)
  - Tailors the prediction horizon to the investment process
- Improved factor structure to better identify sources of risk
- Innovative solution to better estimate *true* factor risk
  - Reduced noise in factor returns & proper allocation of factor/specific risk
- Extensive pre-release testing
  - Three-way reconciliation of model calibration code
  - Extensive model backtesting over multiple horizons
- Enhanced QC process and model governance
  - Monitor model performance daily and conduct formal quarterly review

# MAC3 Volatility vs Implied Volatility (COVID)

- MAC3 exhibits intuitive term structure of risk
  - Daily model is most responsive; annual model is most stable
  - Monotonic decrease in volatility with increasing prediction horizon
- MAC3 is consistent with implied volatilities from options market
- MAC2 responsiveness is between MAC3 Quarterly and Annual



Source: Bloomberg



US Market Portfolio

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# Testing Accuracy of Volatility Forecasts

- Bias statistic
  - Represents ratio of realized risk to predicted risk (ideal value is near 1)
- Q-statistic
  - Most reliable measure of forecast accuracy (penalizes bias and noise)
  - Lower Q-stats imply more accurate risk forecasts ( $\Delta Q=0.01$  is considered significant)

Global Model	B-stat					Q-stat				
	Day	Week	Month	Quarter	Year	Day	Week	Month	Quarter	Year
MAC3 Daily	1.017	1.097	1.209	1.352	1.628	2.453	2.463	2.563	2.760	3.281
MAC3 Weekly	0.972	1.023	1.113	1.236	1.491	2.478	2.422	2.455	2.575	2.966
MAC3 Monthly	0.948	0.983	1.054	1.160	1.382	2.535	2.446	2.431	2.496	2.760
MAC3 Quarterly	0.929	0.954	1.013	1.107	1.300	2.603	2.493	2.449	2.481	2.651
MAC3 Annual	0.773	0.780	0.815	0.883	1.001	2.894	2.749	2.647	2.601	2.544
MAC3 Long-Term	0.780	0.783	0.809	0.859	0.968	2.994	2.843	2.726	2.649	2.557
MAC2	1.083	1.132	1.201	1.292	1.502	2.697	2.655	2.660	2.724	3.020

*Example:  
MAC3 Global Equity Model  
Pure Factor Portfolios*

*Sample Period:  
31-Dec-1999 to 20-Jul-2020*

- MAC3 model aligned with the prediction horizon produced the most accurate risk forecasts

# Outline

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- MAC3 model suite
- Factor structure
  - Improved and expanded set of style factors
  - Use of industry/country beta factors for greater explanatory power
- Factor returns
  - More accurate estimates of factor returns
  - Mitigating spurious correlations between factor/specific returns
- Term structure of volatility
  - Predicting volatility across different horizons
- Factor correlations
  - Applications to risk forecasting and portfolio construction
- Finite-sample adjustment
  - Improved model specification (properly disentangle factor/specific risk)
- Summary

# MAC3 Model Suite

# MAC3 Suite of Equity Models

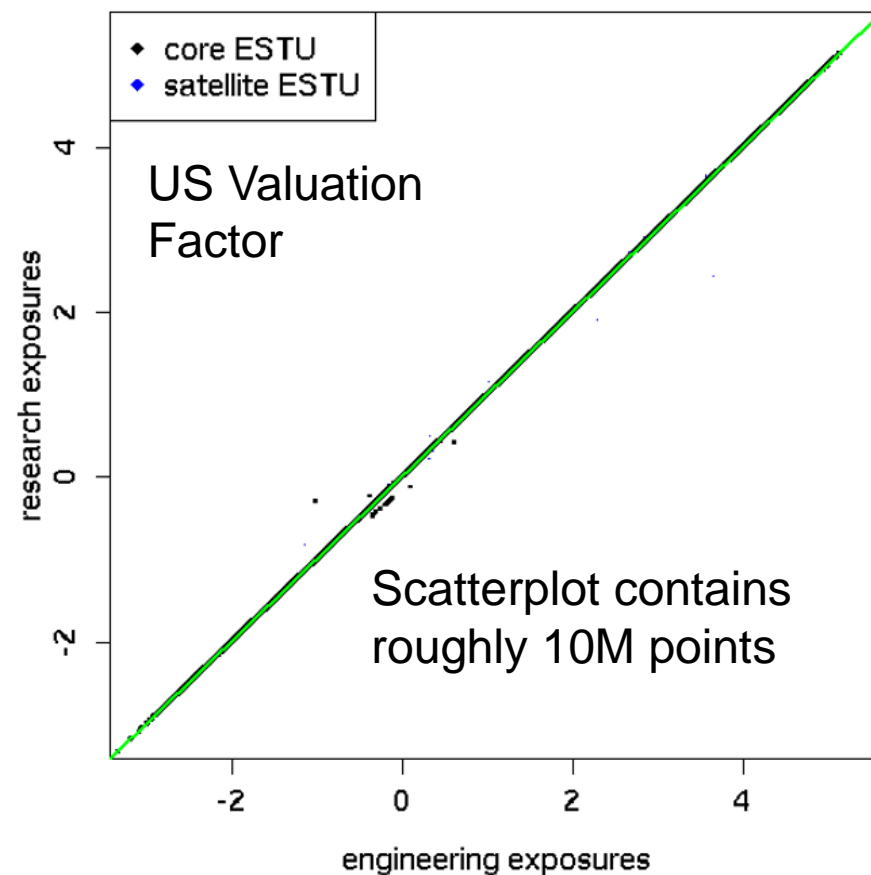
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- Model selection: one global model or 13 “local” models
  - Global
  - Japan
  - **Korea (New)**
  - Asia
  - China A-shares
  - **UK (New)**
  - Europe
  - US
  - Canada
  - Australia/NZ
  - **India (New)**
  - **South Africa (New)**
  - Emerging EMEA
  - Latin America
- Global model spans entire world
  - Estimates portfolio risk using one set of global factors
  - Factor portfolios are globally diversified
- Integrated model spans entire world
  - Aggregates local factors across all 13 local models
  - Allows for more granular description of portfolio risk
- Individual local models
  - Exposure to local factors of only one model (not universal coverage)
  - Extended coverage by means of satellite country factors

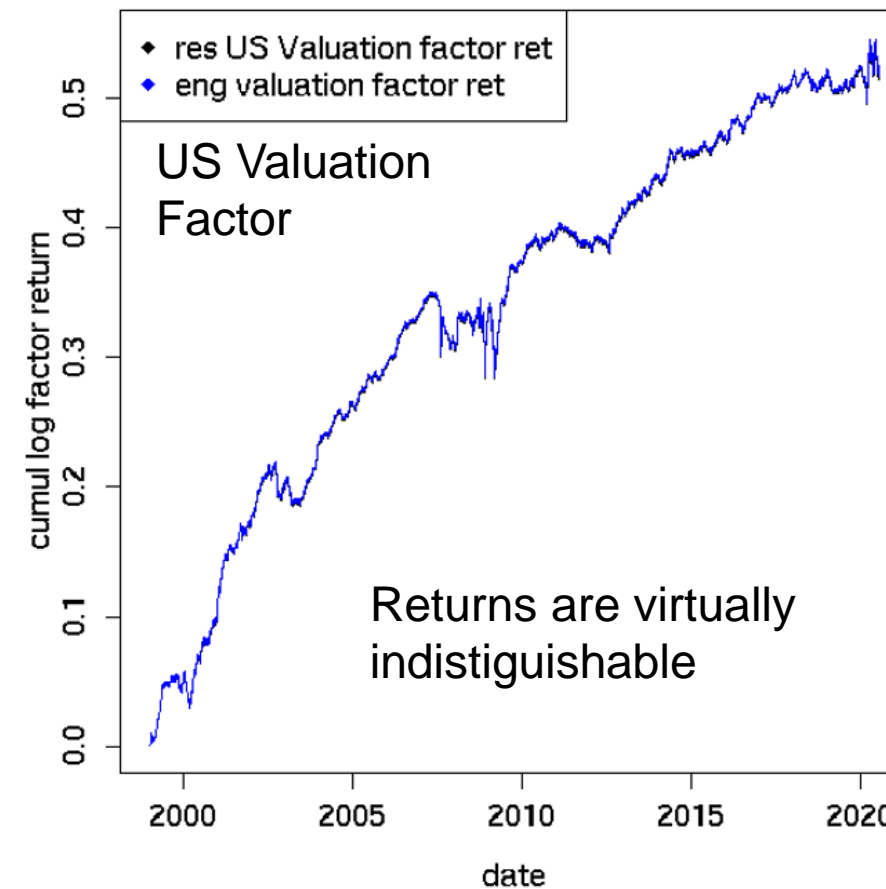
# Independent Validation of Production Code

- For quality assurance, we developed *three* independent codes
  - One code was written by software engineers for production environment
  - Two other codes were written by research team in different languages
  - Typical correlation between research/production was well above 99.99%

## Factor Exposures



## Cumulative Returns





# Factor Structure

# Style Factors

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- MAC3 equity model suite shares a common set of 14 style factors

## Technical Factors:

- Market Beta (New)
- Residual Volatility (New)
- Momentum
- Long-term Reversal (New)
- Size (log of market cap)
- Mid-cap (New)
- Liquidity (STO, BAS, AMI)

## Fundamental Factors:

- Dividend Yield (Indicated D/P)
- Earnings Yield (FEP and HEP) (New)
- Valuation (B/P, S/P, CF/P)
- Profit (ROA, ROE, PRM)
- Growth (HSG, HEG, MTG, LTG)
- Variability (VNI, VSA, VCF)
- Leverage (D/A, D/B, D/M)

- Style factor enhancements (versus MAC2)
  - MAC2 Volatility factor split into Market Beta and Residual Volatility
  - MAC2 Value factor split into Earnings Yield and Valuation
  - Long-term Reversal: Trailing 4y return with most recent year excluded
  - Mid-cap factor: mid-caps have positive exposure, other stocks negative
- Style factors standardized as z-scores (mean zero, unit std)

# Industry Betas

- Risk models typically use industry dummies, i.e., (0,1) exposures
- MAC3 model uses industry betas
  - Large increase in explanatory power of model
  - Improved model specification (mitigate spurious factor/specific correlations)
- Industry betas are standardized to be cap-weighted mean 1
- Multi-country models employ country betas as exposures
- Using (0,1) exposures, factor/specific returns are correlated
- Compute correlation of specific returns of industry beta quintiles with industry factor returns

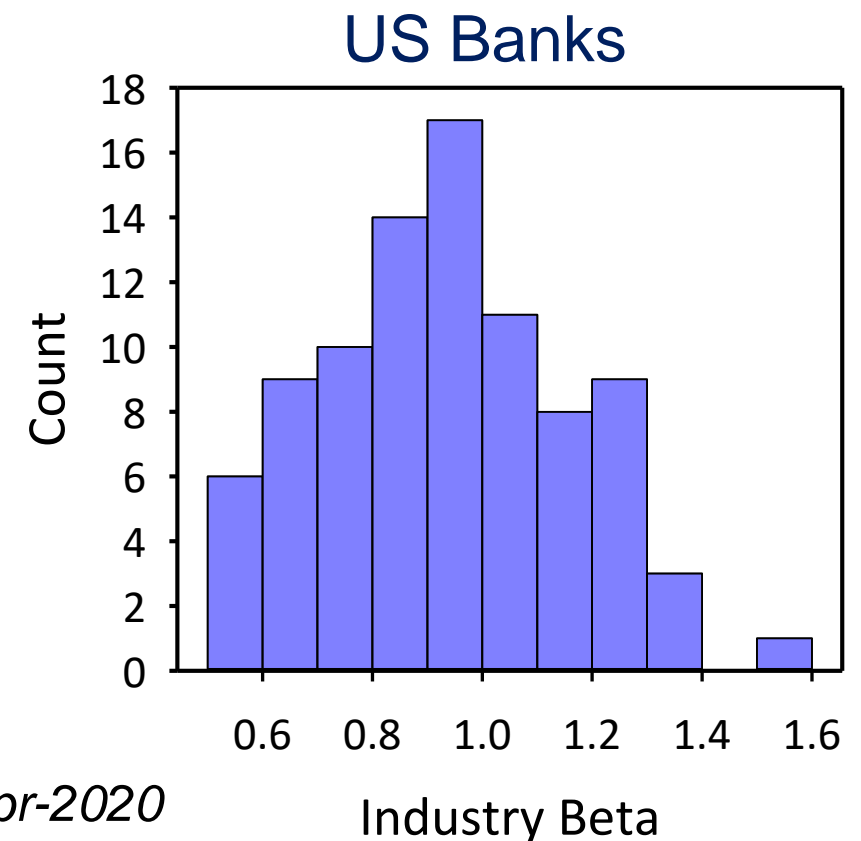
$$r_{nt} = \beta_{ni} R_{it} + e_{nt}$$

Industry Beta

Factor/specific correlations:

Quintile	Betas	Dummies
Top	0.024	0.245
Bottom	-0.188	-0.416

*Averaged across all US industries*



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# Factor Returns

# Factor Returns and Pure Factor Portfolios

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- Output of regression is a set of *pure factor portfolios*
- Market factor portfolio is essentially the cap-weighted ESTU
- Country factor portfolios (strictly dollar neutral)
  - Long a portfolio with unit exposure to the country; short the market
  - Industry and style neutral
- Industry factor portfolios (strictly dollar neutral)
  - Long a portfolio with unit exposure to the industry; short the market
  - Country and style neutral
- Style factor portfolios (strictly dollar neutral)
  - Unit exposure to style in question
  - Country and industry neutral (also neutral to all other styles)
- MAC3 employs more-efficient regression weights
  - Traditional approach uses square-root-of-market-cap; MAC3 uses inverse residual variance (IRV)
  - MAC3 approach cuts “noise” by roughly 30% relative to root-cap

# Impact on Factor/Specific Correlations

- The *estimated* factor/specific returns should be uncorrelated
  - This is the firm belief and expectation of risk-model users
- Mathematically, pure factor/specific portfolios are uncorrelated only if we use inverse-specific-variance regression weights
- Build two risk models: (1) IRV weights, and (2) root-cap weights
- Use MVO to form min-vol portfolio with unit exposure to each style
  - Both models claim to have produced the true min-vol style portfolio
  - In reality, IRV model produced lower out-of-sample volatility (by 7%)
  - IRV model translates into higher risk-adjusted performance
- Root-cap model also produces spurious correlations between factor/specific returns
- IRV weights mitigate spurious correlations

## Out-of-Sample Volatilities

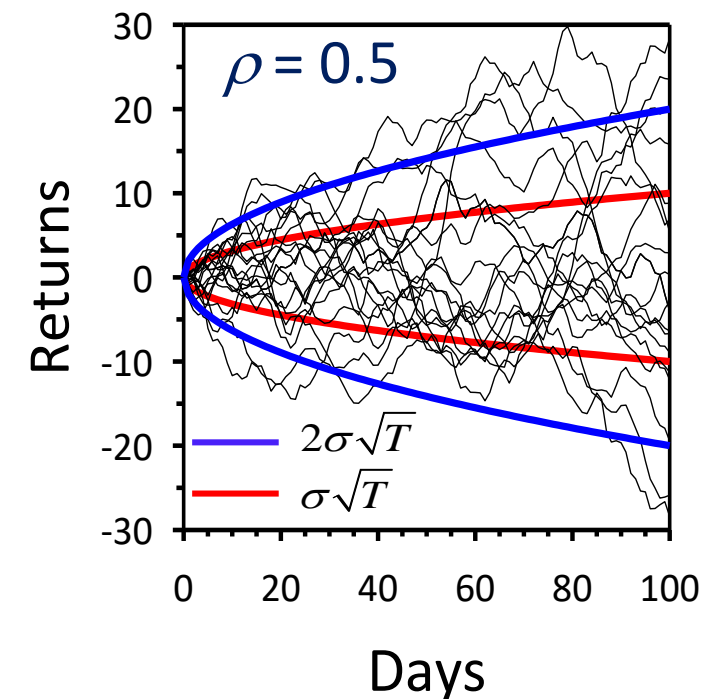
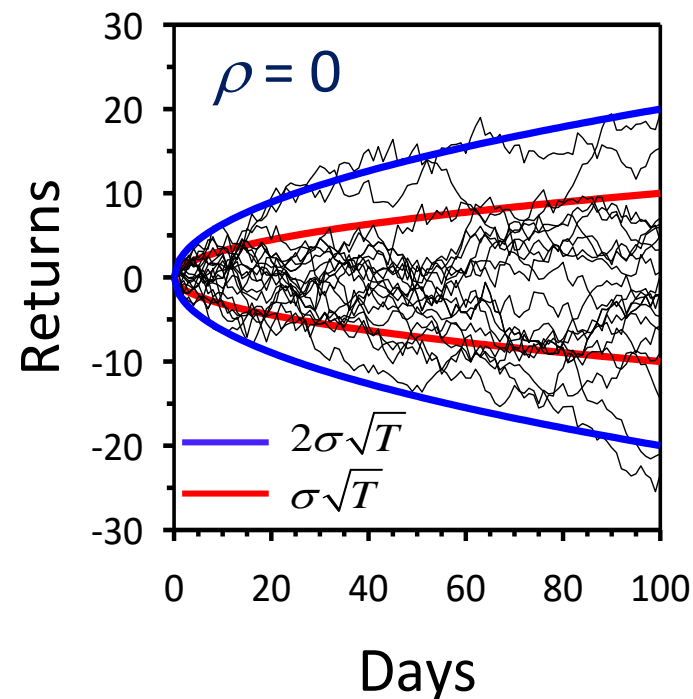
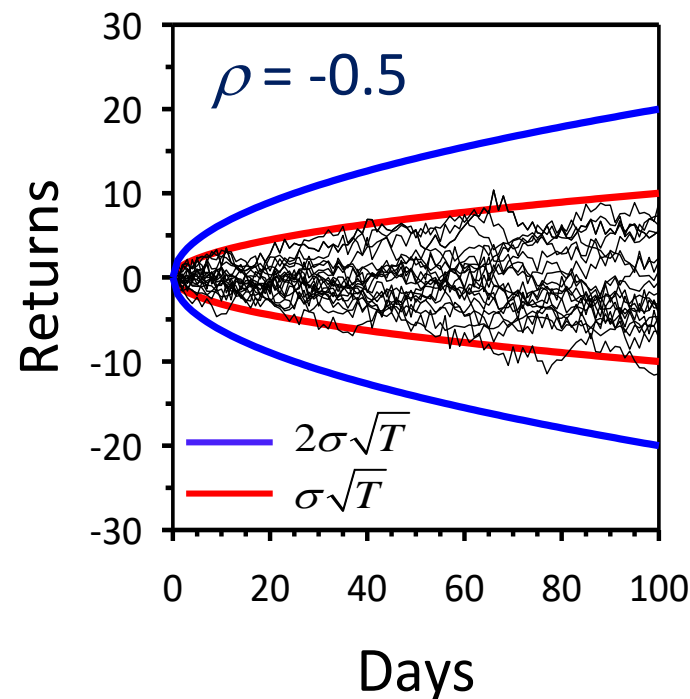
Quantity	Regression Weights		Ratio RC/IRV
	Inv. Res. Var.	Root-cap	
Total Volatility	1.98%	2.12%	1.07
Factor Volatility	1.82%	2.16%	1.19
Specific Volatility	0.77%	1.33%	1.73
Correlation	-0.03	-0.35	11.67

*MAC3 US Model (Jan-2000 to Jul-2020)  
Daily model and daily rebalance*

# Term Structure of Volatility

# Importance of Serial Correlation

- Root-time scaling ( $\sqrt{T}$ ) is valid under zero serial correlation
  - Assumes returns from one period are independent of other periods
- If serial correlation exists, root-time scaling is no longer valid
  - Negative serial correlation acts to reduce  $T$ -period volatility
  - Positive serial correlation acts to increase  $T$ -period volatility
- Simulate 20 paths of portfolio returns with unit volatility ( $\sigma=1$ )
  - Vary serial correlation from -0.5, 0.0, and 0.5





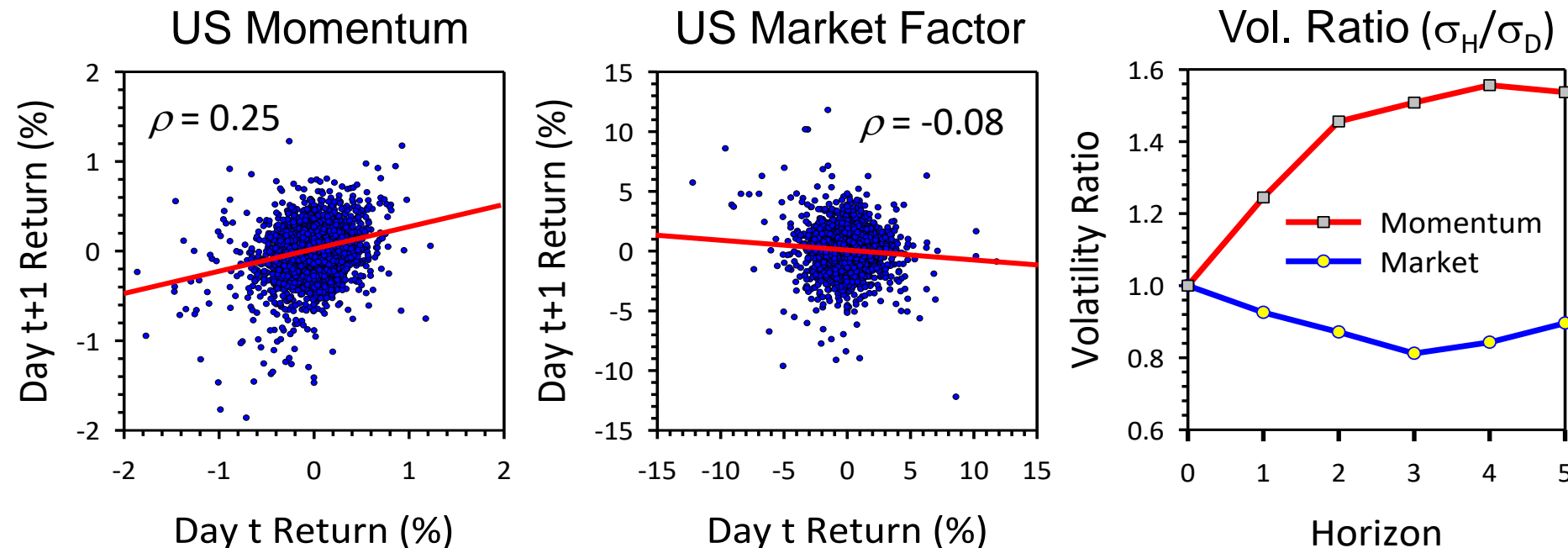
# Example: US Momentum and Market Factors

- Serial correlation of daily factor return (1d lag)
  - Momentum factor exhibits positive correlation (0.25)
  - Market factor shows negative serial correlation (-0.08)
- Effect of serial correlation
  - Momentum volatility increases at longer horizons
  - Market factor volatility decreases at longer horizons
  - Volatility ratio:  $T$ -period volatility divided by daily volatility (annualized)

## Annualized Volatility

Horizon	US MKT	US MOM
Day	19.56	3.43
Week	18.11	4.27
Month	17.04	4.99
Quarter	15.89	5.17
Half-Year	16.49	5.34
Year	17.53	5.27

31-Dec-1998 to 30-Apr-2020



# Estimating $T$ -Period Factor Variance

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- Newey-West Estimate
  - Use high-frequency observations (daily) to forecast risk over longer horizons
  - Explicitly estimates correlation of factor returns across different days
- Low-frequency estimate
  - Aggregate factor returns across  $T$ -days
  - Automatically incorporates effect of serial correlation
  - Compute volatility of rolling  $T$ -day returns
- Apply CSV adjustment to each component (described below)
- Blend the two variance estimates (weighted average)
- Benefits of blending two estimates
  - Typically leads to more accurate forecasts (reduces estimation error)
  - Common example: taking an average of polls for greater accuracy
  - Basically represents “diversification” of error

# Benefits of Blending

- Consider two noisy/biased variance estimates ( $\sigma_T^2 = 1$ )

$$\hat{\sigma}_1^2 = 1 + \mu_1 + \varepsilon_1 \qquad \hat{\sigma}_2^2 = 1 + \mu_2 + \varepsilon_2$$

- Blending the two forecasts may reduce estimation error

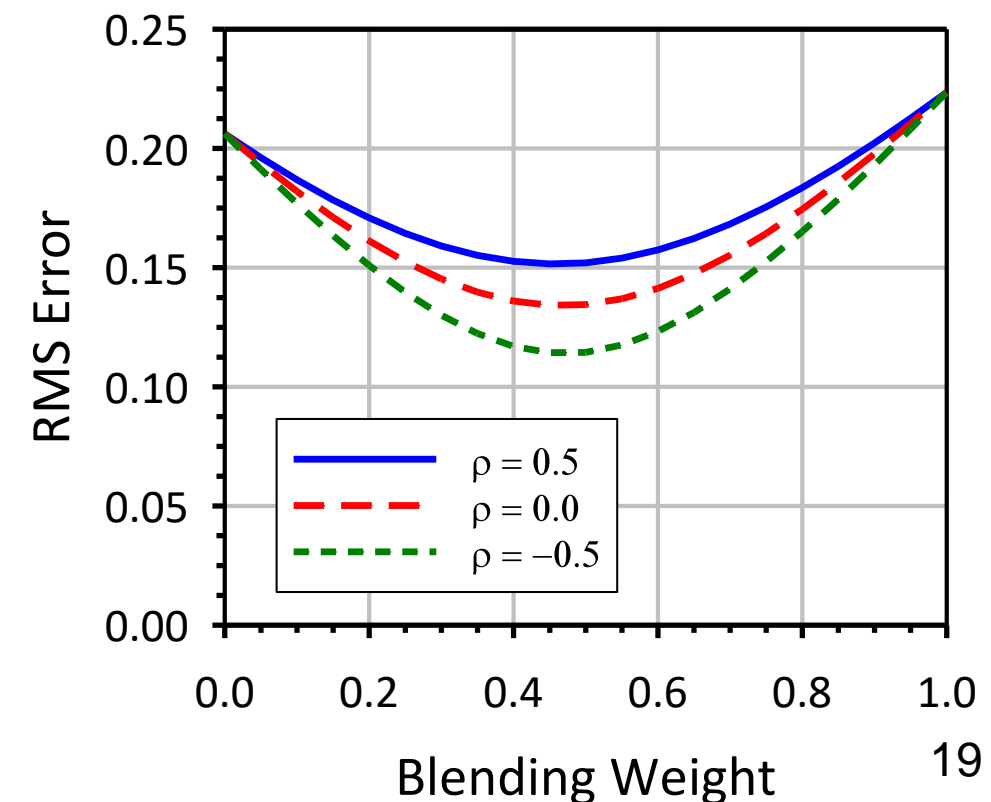
$$\boxed{\hat{\sigma}_B^2 = w_1 \hat{\sigma}_1^2 + (1 - w_1) \hat{\sigma}_2^2} \quad \text{Blended Variance}$$

- Plot RMS error vs blending weight
- Blending reduces RMS error
  - Blended estimate has smaller error than either of the two estimates
  - Reduction in error is even larger if the error terms are negatively correlated
  - Significant error reduction even if errors are positively correlated

*Noise and bias parameters*

Error Type	Estimate 1	Estimate 2
Bias ( $\mu$ )	0.20	-0.05
Noise ( $\varepsilon$ )	0.10	0.20

$$RMS = \sqrt{E\left[\left(\hat{\sigma}_B^2 - 1\right)^2\right]}$$



# Cross-Sectional Volatility (CSV) Adjustment

- Volatility estimation involves finding optimal balance between:
  - Using a short HL to give most weight to recent observations
  - Using a long HL to minimize sampling error
- Using cross-sectional data allows for more weight on recent observations without incurring high penalty in sampling error
- Cross-sectional bias statistic identifies “instantaneous” bias

$$B_t^2 = \frac{1}{K} \sum_k \frac{f_{kt}^2}{\sigma_{kt}^2} \quad \text{Cross-sectional Bias Statistic (Period } t\text{)}$$

- Compute mean bias over recent periods (EWMA)

$$B^2 = \sum_t v_t B_t^2 \rightarrow \boxed{\tilde{\sigma}_k = B \sigma_k} \quad \text{CSV Adjusted Volatility}$$

- Low sampling error (2d HL with 50 factors  $\approx$  300 observations)

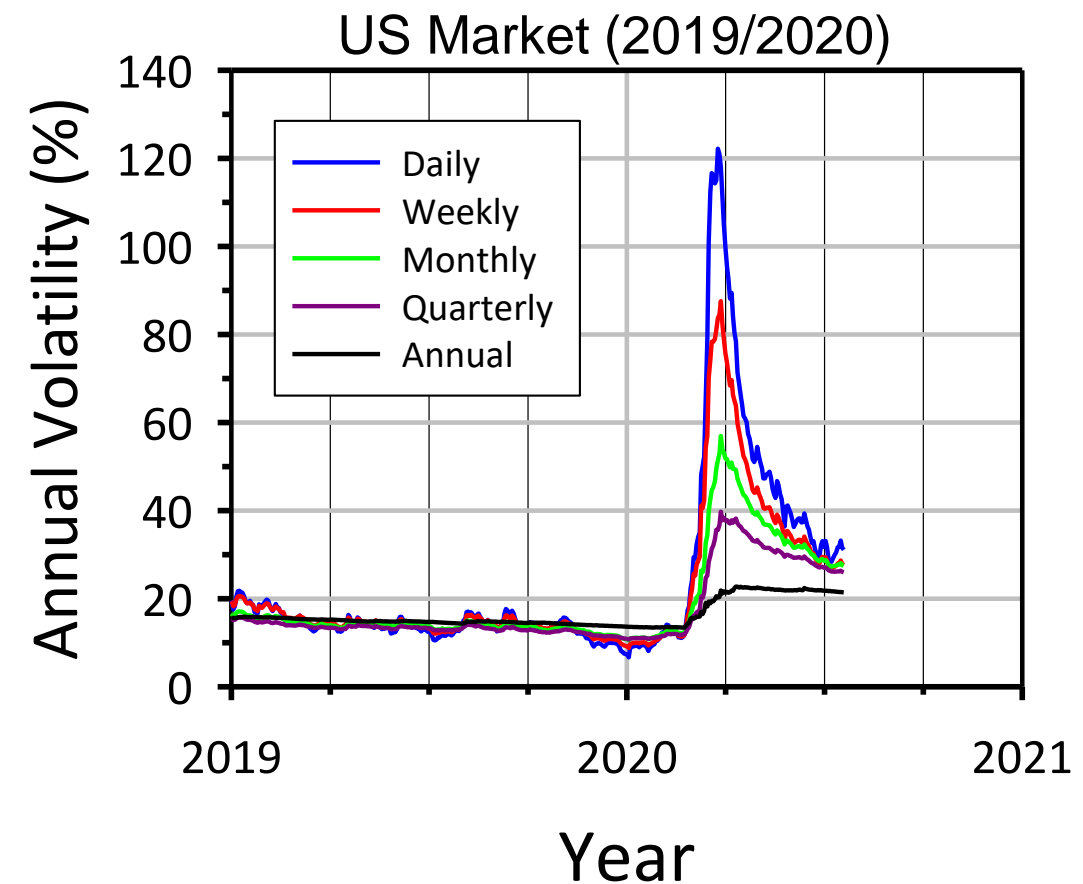
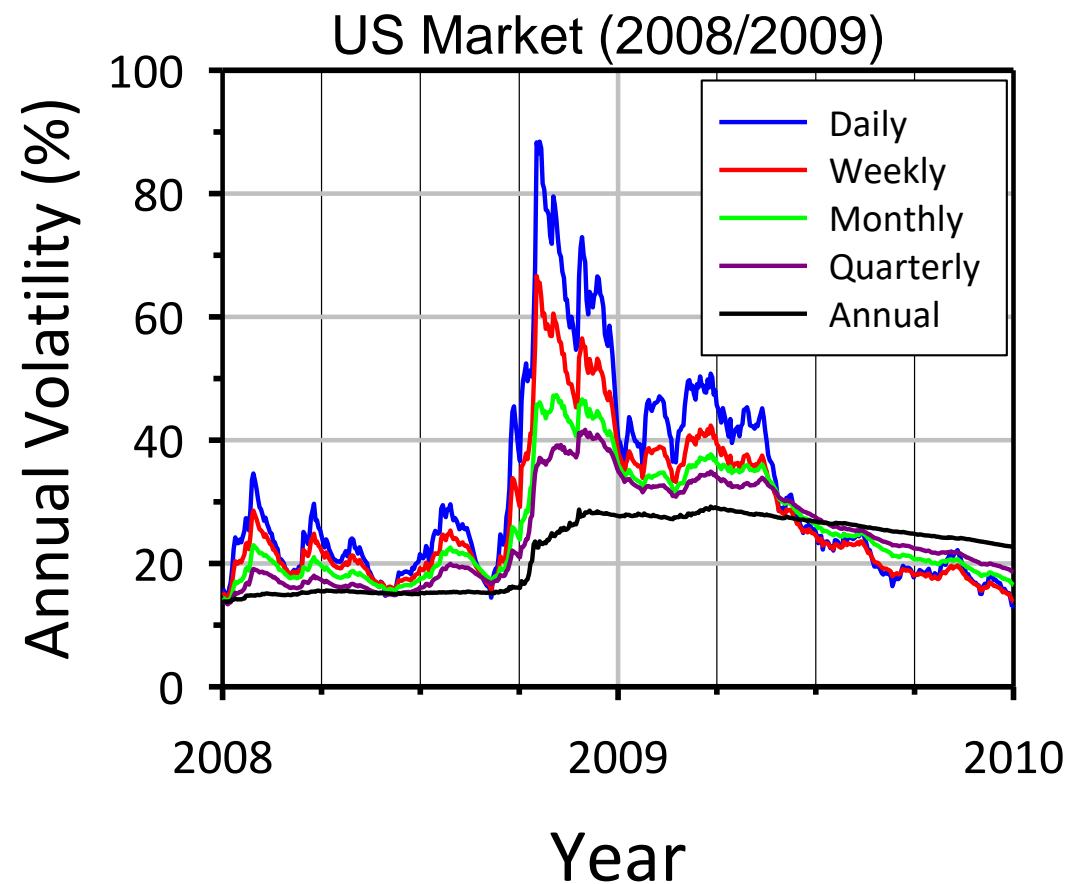
Menchero and Morozov. *Improving Risk Forecasts Through Cross-Sectional Observations*,  
Journal of Portfolio Management, Spring 2015, pp. 84-96

# Term Structure of Volatility

- Mixed-frequency (MXF) CSV
  - We find significant improvement in risk forecasts by applying separate CSV adjustments to the high-frequency and low-frequency components

$$\hat{\sigma}^2 = w_{NW} B_{NW}^2 \hat{\sigma}_{NW}^2 + (1 - w_{NW}) B_{LF}^2 \hat{\sigma}_{LF}^2$$

MXF CSV Variance



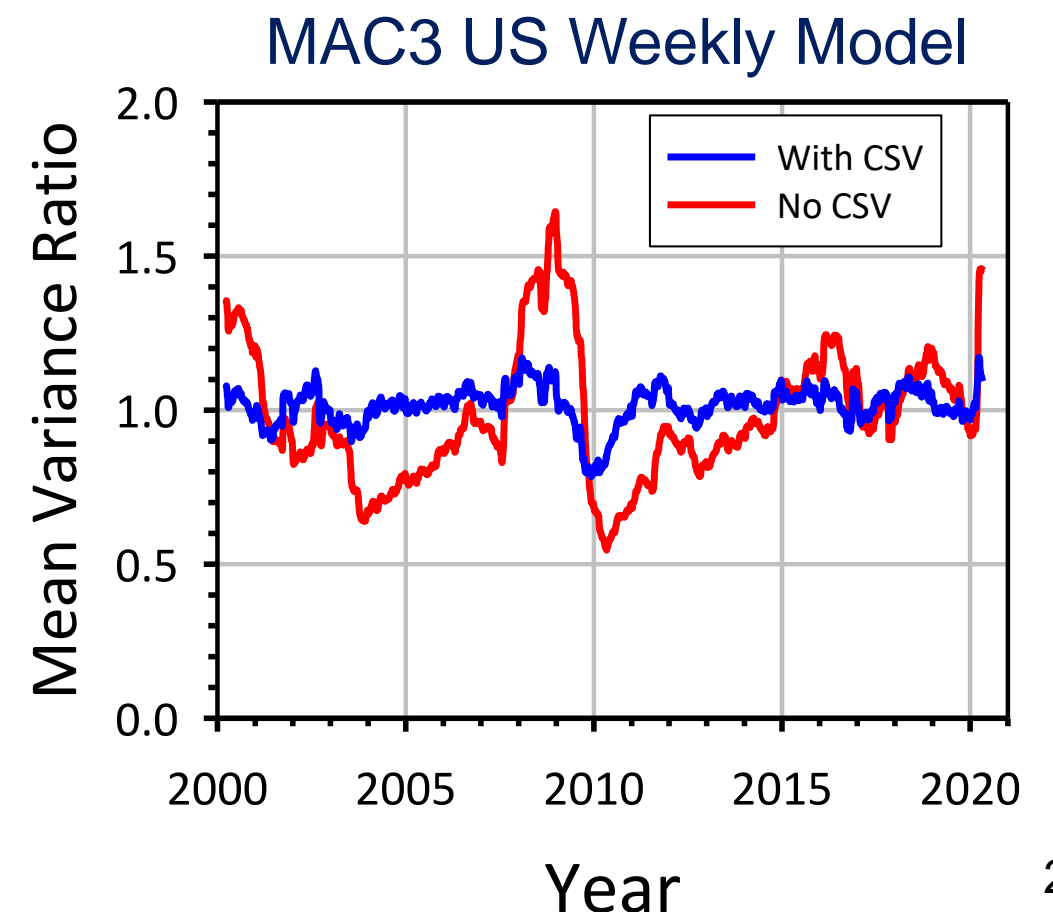
# CSV in Action: Mitigating Biases

- Variance Ratio represents realized-to-predicted variance

$$z_{kt} = \frac{f_{kt}}{\sigma_{kt}} \rightarrow \boxed{B^2 = \frac{1}{KT} \sum_{kt} z_{kt}^2}$$

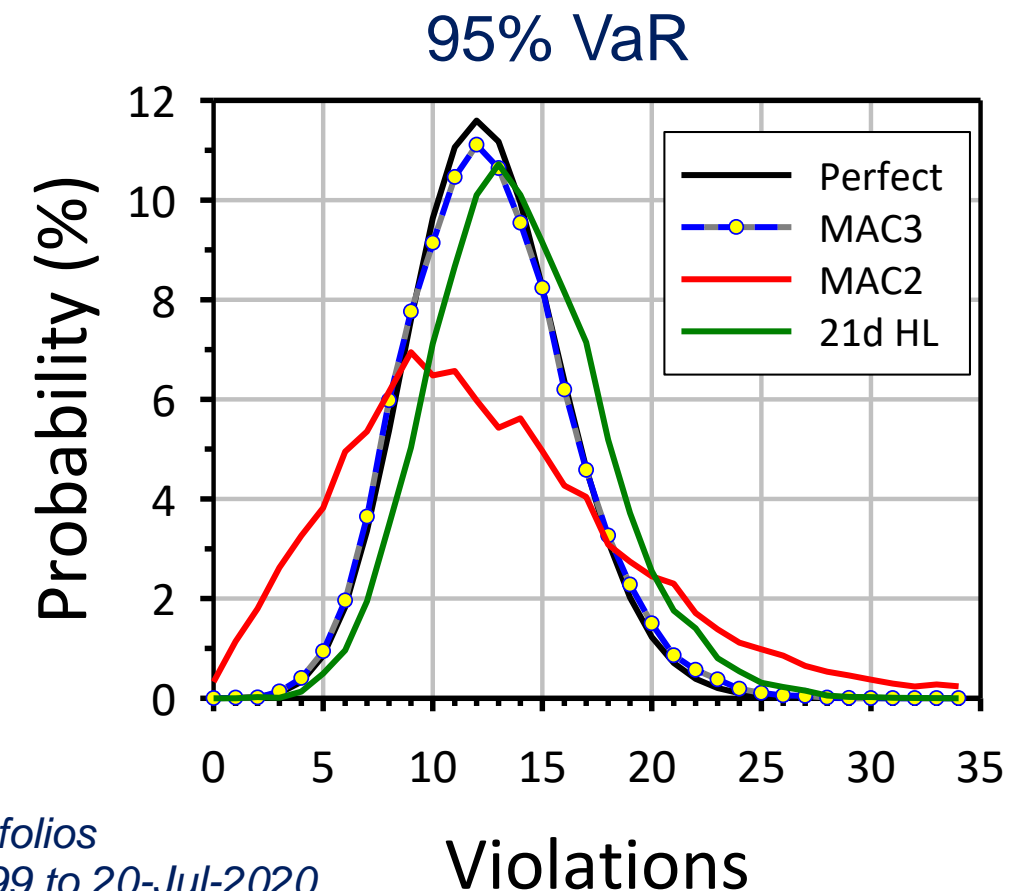
Variance Ratio (Squared B-stat)

- Compute trailing 252-day variance ratio across all factors
- For perfect risk forecasts,  $E[B^2] = 1$
- Compare variance ratios with and without CSV
  - Models are identical in every other respect
- Model without CSV led to biased risk forecasts
  - Significant underforecasting *during* financial crises
  - Significant overforecasting *following* financial crises
- CSV technique greatly mitigates biases in risk forecasts
  - CSV forecasts adapt very quickly to changing market regimes



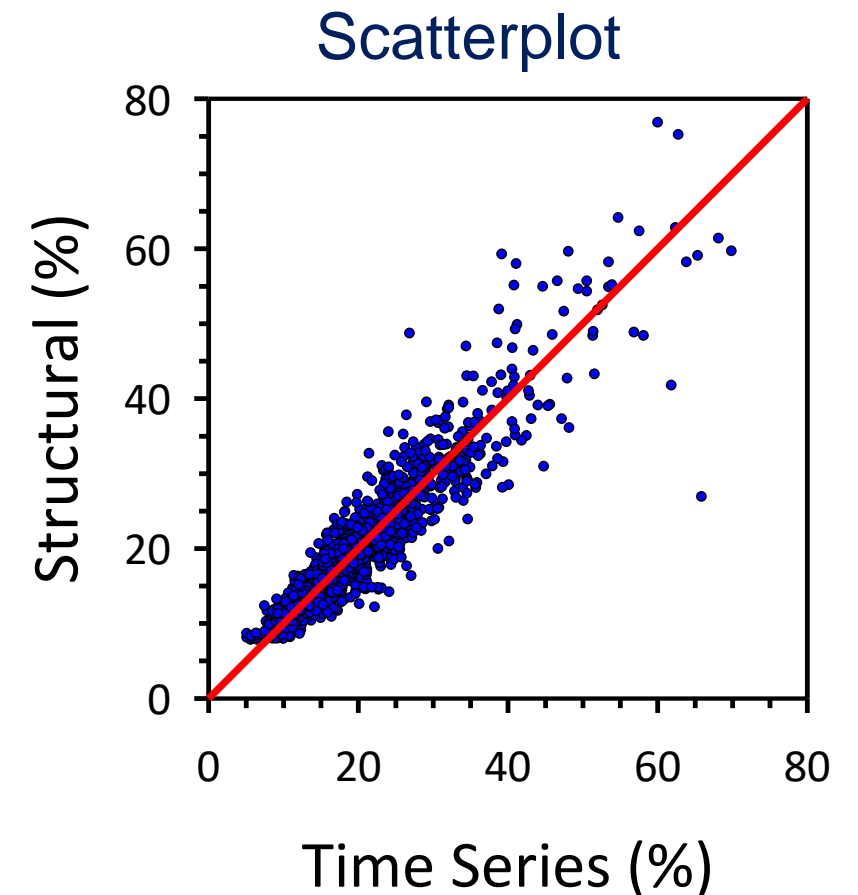
# Testing Accuracy of Tail-Risk Forecasts

- Perfect VaR forecasts:
  - The expected number of violations for 95% VaR is 12.5 (250-day windows)
  - Probability of observing a given number of violations follows binomial distribution
  - Compare empirical distribution with distribution using perfect forecasts
- MAC2 Model
  - Low responsiveness causes violations to pile up on both ends
  - Extended periods of overforecasting or underforecasting
- 21-day HL Model
  - Bias (caused by insufficient responsiveness) causes too many instances of finding a large number of violations
  - Noise causes too few instances of finding few violations
  - By chance, VaR will often be too low leading to violations
- MAC3 Model
  - Matches theoretical distribution almost perfectly



# Specific Risk Model

- Compute specific variance using time series of specific returns
  - Apply same methods as used for factor variance
  - Blend NW forecast with low-frequency estimate
- Some stocks lack reliable time series (e.g., IPOs)
- We need a *structural* model for such stocks
- Regress time-series estimates against the factor exposures
- Intuition: specific risk can be largely explained by the stock's factor exposures, e.g.,
  - Stocks with high residual volatility tend to have high specific risk
- Blend the time-series forecast with the structural forecast
  - Blending the two estimates is effective at improving the accuracy of risk forecasts



US Monthly Model (31-Dec-2019)



# Factor Correlations

# Estimating $T$ -Period Factor Correlations

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- MAC3 factor correlations are estimated similar to factor volatilities
  - Blend Newey-West estimate with low-frequency estimate
  - Represents our best estimate of “sample” correlation at given horizon
- Sample correlation is unsuitable for portfolio optimization
- Naïve shrinkage (toward zero) is beneficial from a portfolio construction perspective

Menchero, Jose and Lei Ji. *Portfolio Optimization with Noisy Covariance Matrices*, Journal of Investment Management (2019)

- Sample correlation is essentially optimal for risk forecasting
- Naïve shrinkage is very detrimental to the accuracy of risk forecasts

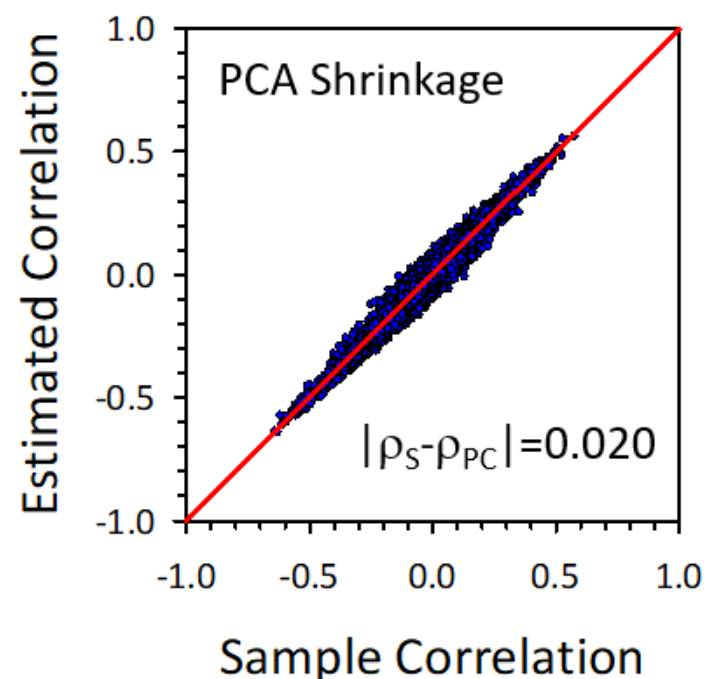
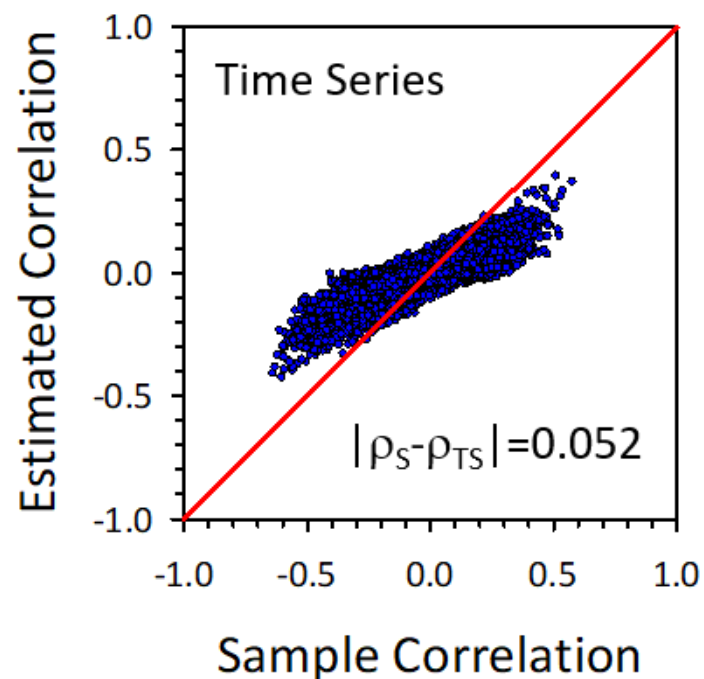
Menchero, Jose and Peng Li. *Correlation Shrinkage: Implications for Risk Forecasting*, Journal of Investment Management (2020)

- Goal: find robust correlation matrix that deviates minimally from sample correlation
- MAC3 solution: PCA Shrinkage (blend sample with PCA correlation)

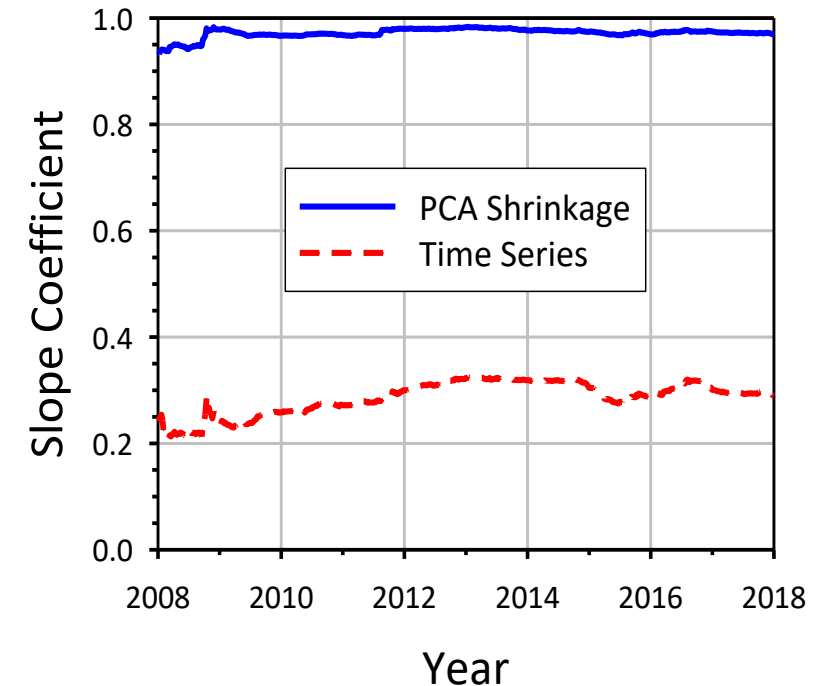
Menchero, Jose and Lei Ji. *Advances in Estimating Covariance Matrices*, Journal of Investment Management (to appear 2021)

# Correlation Scatterplots

- Dual objectives:
  - Produce well-conditioned correlation matrix suitable for portfolio optimization
  - Deviate minimally from sample correlation for accurate risk forecasts
- Conventional approach uses the “time-series method”
  - Try to identify “global” factor returns to explain “local” correlations
- Time-series method tends to systematically underforecast  $\rho$
- PCA shrinkage deviates minimally from the sample



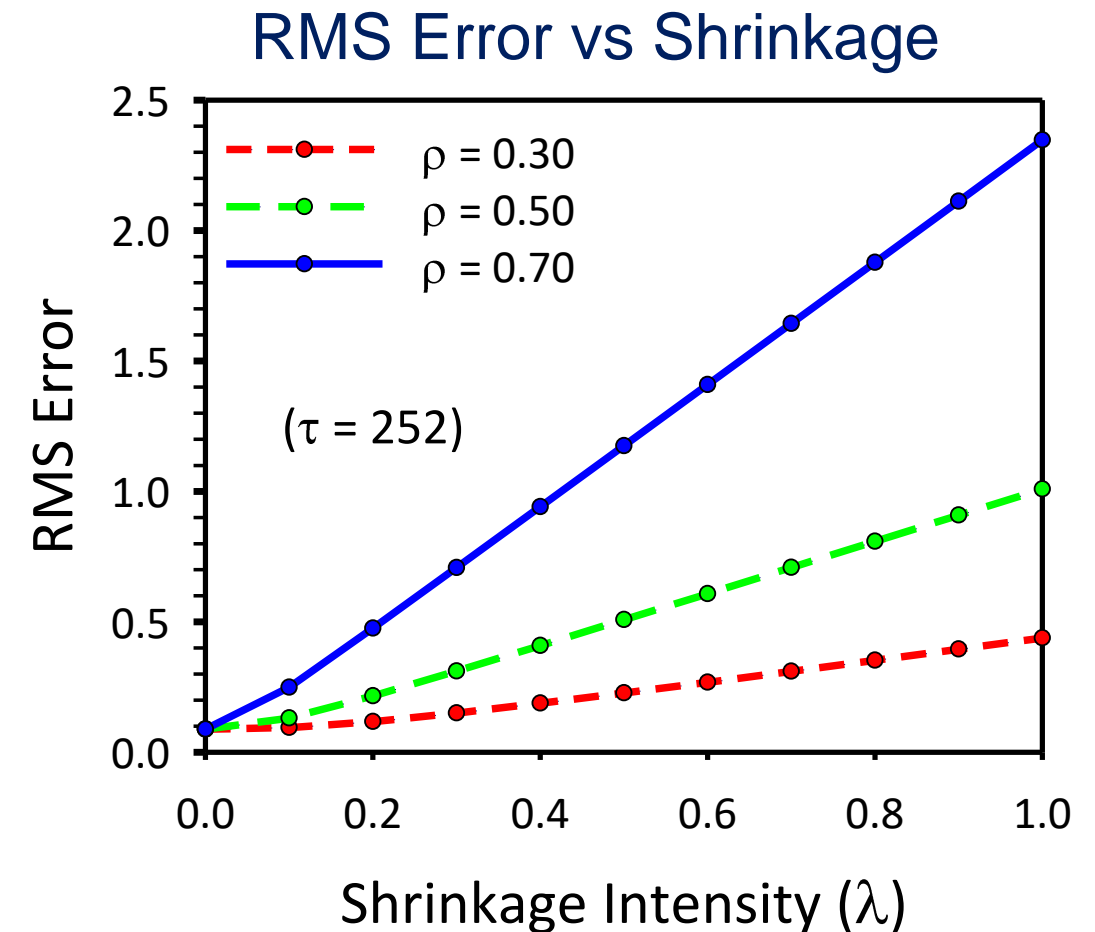
*Example: correlation between equity factors and fixed income factors*



Menchero, Jose and Lei Ji.  
*Advances in Estimating Covariance Matrices*, Journal of Investment Management (2021)

# The Sample Correlation is Nearly Optimal

- Predict risk for two-asset portfolio:
  - Go long one asset and go short another asset
- Plot RMS error versus shrinkage intensity
  - Consider three values for true correlation: (0.30, 0.50, and 0.70)
  - Consider a 252-day look-back window for estimation
- Zero shrinkage appears optimal
  - Actually, shrinkage always reduces RMS error
  - The error reduction is just so tiny that it is not visible to the naked eye
  - Excess shrinkage induces large errors in risk forecasts
  - Forecasting error is exacerbated as the correlation and/or shrinkage intensity increases
- Effectively, the sample correlation is *optimal* for predicting risk



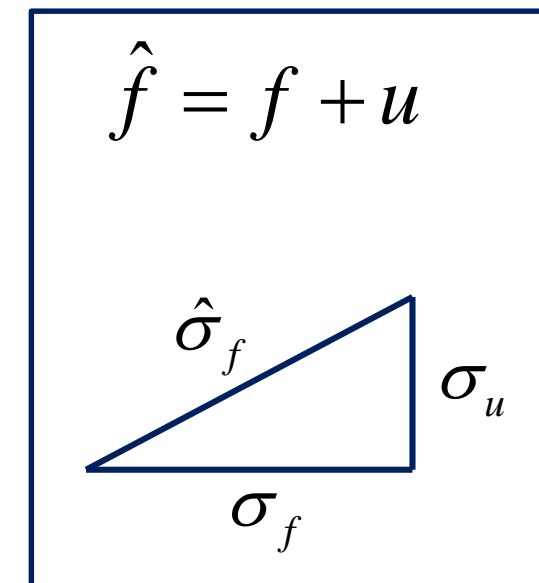
Menchero, Jose and Peng Li. *Correlation Shrinkage: Implications for Risk Forecasting*, Journal of Investment Management (2020)

# Finite-Sample Adjustment

# Finite-Sample Adjustment

- Factor models have been misspecified since days of Barr Rosenberg (1970's)
- Failure to distinguish between true (unobservable) factor/specific returns and their *estimated* values (observable quantities)
- Pure factor portfolios have unit exposure to their respective factor
  - They are driven by the unobservable “true” factor
  - But they also contain an idiosyncratic component
- Traditional approach uses estimated variance of pure factor portfolio as the “true” factor variance
  - Risk model adds on “another layer” of specific risk
  - Effectively “double counts” specific risk of pure factors
  - Leads to overforecasting risk of pure factor portfolios
- FSA also corrects under-forecasting of specific risk
  - Extreme example: an industry with a single stock would have zero specific risk in the traditional approach

True vs Estimated

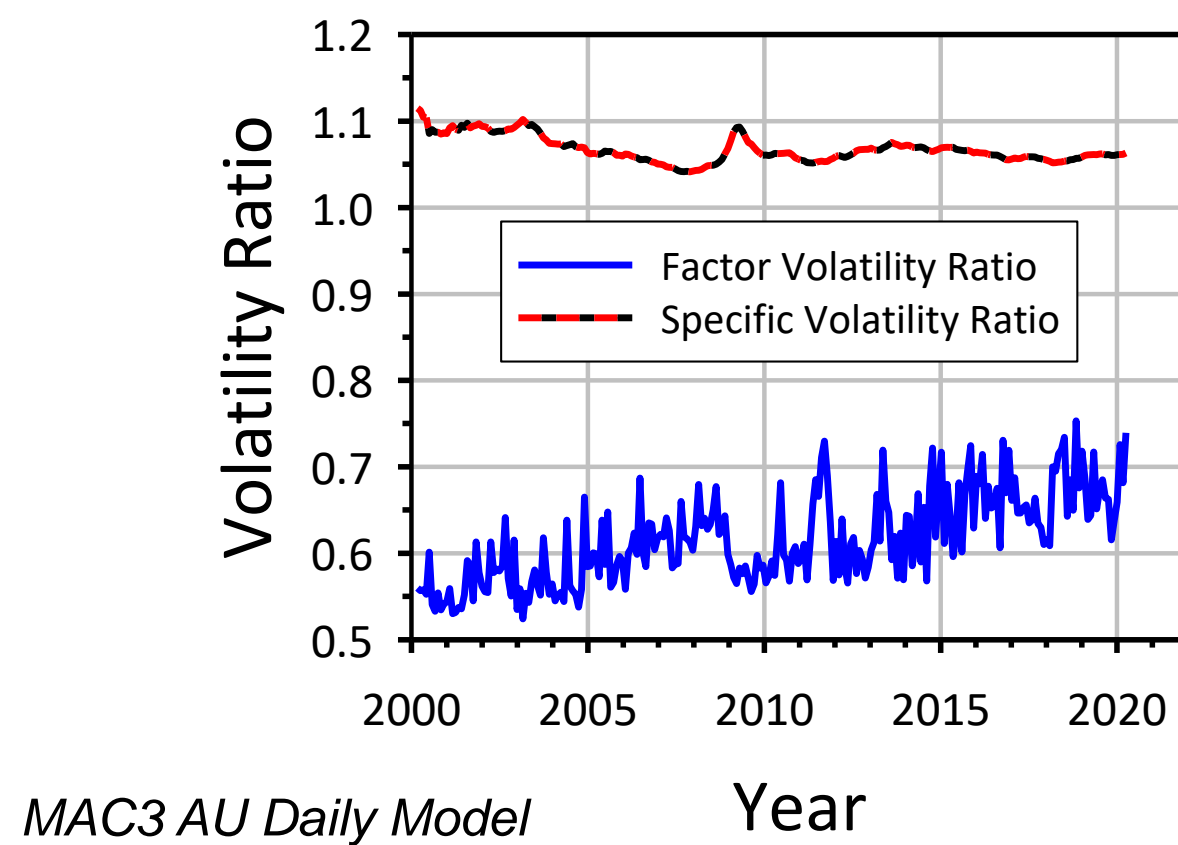
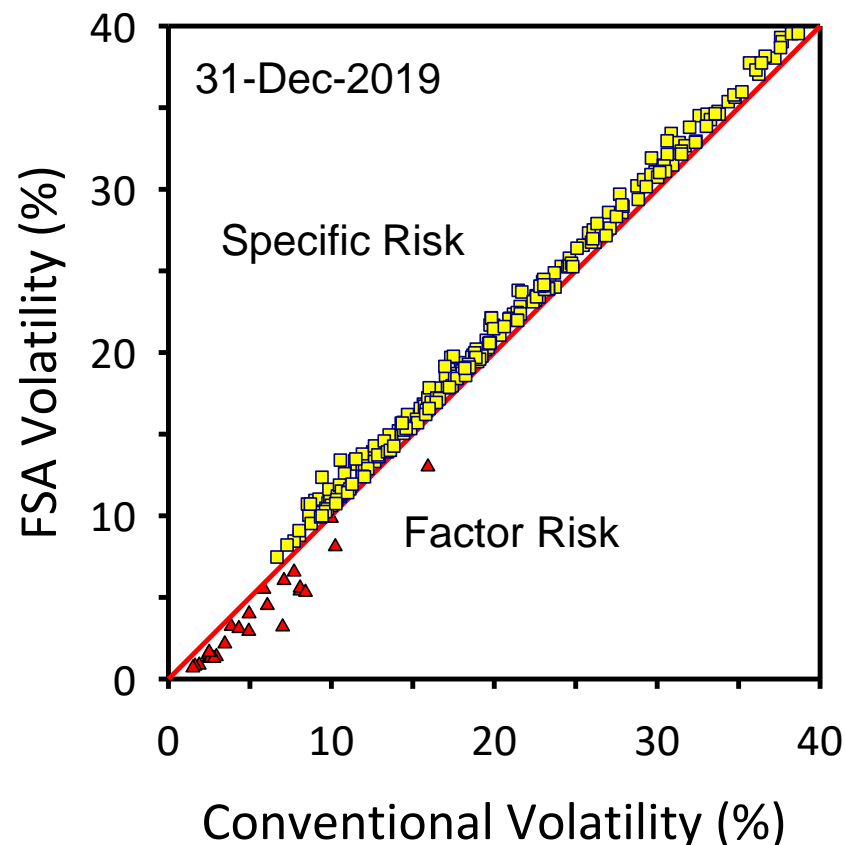


# The Effect of FSA (AU Model)

- MAC3 model “disentangles” factor risk from specific risk:
  - FSA factor volatility ratio is 0.62 on average (AU Model)
    - Large for strong factors like Market (0.97), smaller for weak factors
  - FSA specific volatility ratio is 1.07 on average (AU Model)
    - Larger ratio for stocks with large regression weights in thin industries

$$v_F = \frac{1}{K} \sum_k \frac{\sigma_k^{FSA}}{\sigma_k^{Conv}}$$

$$v_S = \frac{1}{N} \sum_n \frac{\sigma_n^{FSA}}{\sigma_n^{Conv}}$$

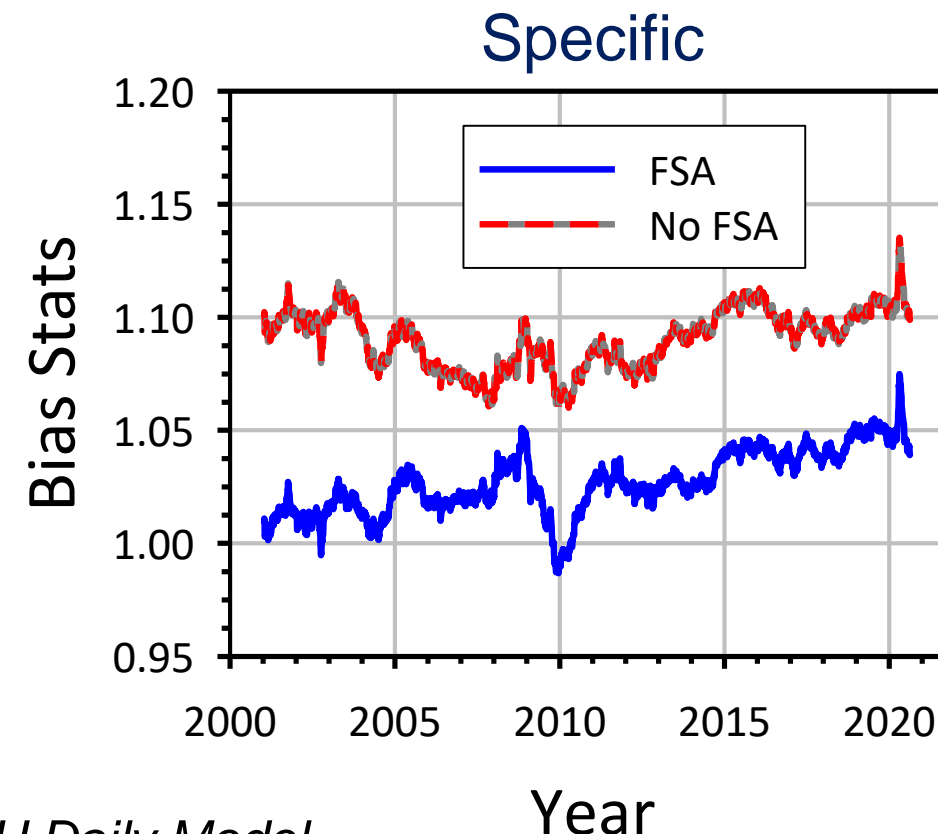
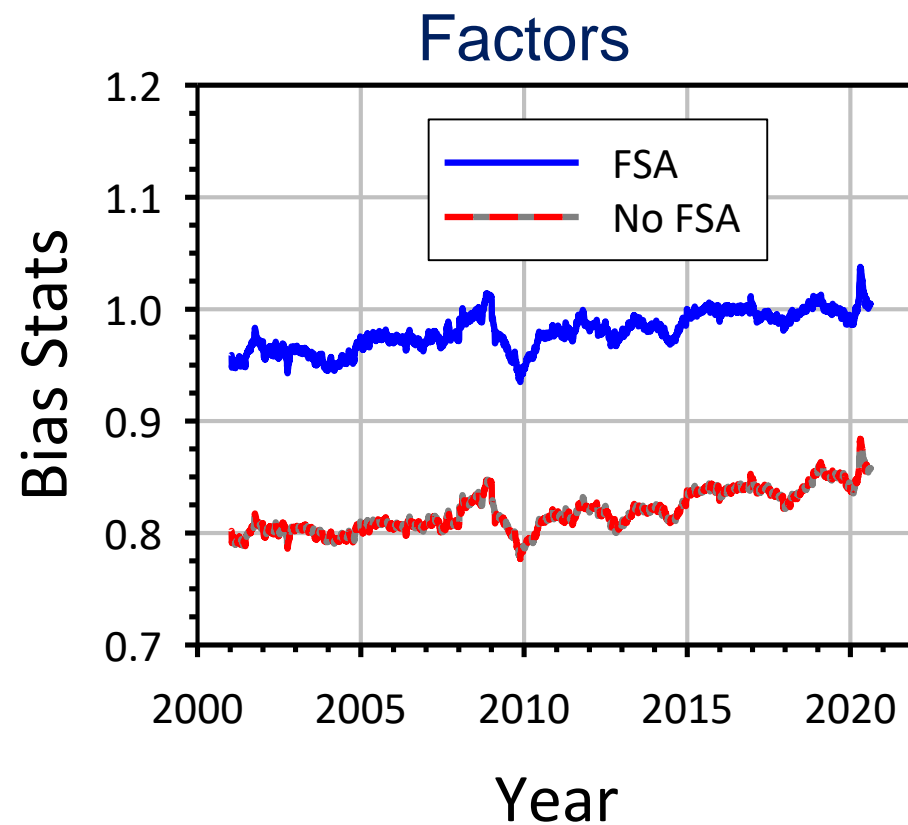


# Bias Statistic Comparison

- Compute mean trailing 252-day bias stats
  - Compute with & without FSA (all else equal)
  - FSA bias-stats consistently closer to 1
  - No-FSA persistently overforecasts factor risk
  - No-FSA persistently underforecasts specific risk

Mean Bias Statistics

Method	Factor	Specific
FSA	0.98	1.03
No FSA	0.82	1.09



MAC3 AU Daily Model



# Summary

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- **MAC3 model incorporates four years of intensive research:**
  - More granular set of local factor models (four new local models)
  - Satellite factors to capture unique country risk while keeping ESTU “pure”
  - Independent validation of production code to ensure model quality
  - Introduction of new style factors
  - Use of industry/country betas for higher  $R^2$  and better model specification
  - Improved regression-weighting scheme to reduce noise in factor returns
  - Full term structure of risk for volatilities and correlations
  - Extensive use of blending/shrinkage to reduce estimation error
  - Use of cross-sectional observations for more accurate risk forecasts
  - PCA blending for robust portfolio optimization and accurate risk forecasts
  - New specific risk model including structural component
  - Finite-sample adjustment to properly disentangle factor/specific risk
  - Full daily updates of all model components with flat-file delivery
- **MAC3 model will be extended to multiple asset classes (2021)**